Working towards playfulness

About the foundations of learning mathematics.

Les Staves

Contact

email
les.staves @ ntlworld.com

phone
0113 2178685

Website
www,veryspecialmaths.co.uk

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The EQUALS
guide to mathematics

www.equals.co.uk
0191 272 8600

The importance of play in learning
Recognising early progress working towards intentional action
Developing active exploration and learning schema related to mathematical learning .

The importance of play in learning

For the typical child play is a spontaneous activity, it satisfies what seems to be an active will to learn. It is exhibited in the ceaseless curiosity, exploration, manipulation and practice that is part and parcel of the development of the child's body skills, and in developing ideas e.g. Their understanding of shape, texture, weight, quantity etc .which are all related to mathematical understanding.

Before they are able to explore and play children must develop

- sensory skills,
- attention and perceptual awareness,
- manipulation and movement skills

- At first, children's play is solitary manipulation and movement, as time progresses they usually develop towards more social dimensions.
- Initially social play entails playing alongside others, without mixing,
- Later it involves becoming engaged in associative and cooperative play with others, through which the rules of coexistence are experienced. This gives children the chance to develop social competence and supports the development of communication and thinking skills.

These processes underpin the child's abilities to gather information from other people, this includes understanding events, quantities, space and time and as such is intrinsic in their development of social mathematics.

The early years foundation stage recognises the power of play as a motivational driver of learning. It also emphasises the important role of the practitioner, in supporting and extending play through well planned environments and motivating activities, skilful intervention and contributions that promote the Childs will to explore and learn. Staff working with very special pupils will often need to enable their pupils to promote more spontaneous activity in ways that are not necessary for more typically developing children. Staff may also need to maintain such approaches throughout the school life of some pupils. The art of teaching includes balancing direct support and modelling with promoting independent learning, which matures in context as the pupils grow older.

Recognising progress at earliest levels of learning

At birth the typical child immediately begins the processes of exploration that are the driving forces of its development. – seeking food, experiencing touch, beginning to learn about looking and listening. These processes seem to be innate. We take them for granted.

However children born with profound difficulties may not be able to interact with their environments in these active ways, and they may need support to develop
The framework below[i] draws from Brown 1996[ii] and Byers 1996[iii]. It is quoted as an important source in Quest for Learning (Northern Ireland)[iv] 'Routes for Learning' (Welsh Assembly 2006)[v] and it is part of the background to P scales used for assessment in England.

It can help teachers recognise pupil's learning at very early levels

In particular the words in the first column are useful to understanding

- Ways in which pupils engage with their learning environments
- Transitions between levels of development for pupils with profound learning difficulties.

The words outline different **levels of engagement** possible changes in individual pupils' responses and behaviour, as their early perceptions of experiences and increasing involvement in the learning process develop into knowledge, skills and understanding.

The development of internal learning processes, *for example, thinking skill*s, is shown by increasing degrees of attention, discrimination, responses and participation in experiences and activities.

A framework for recognising progress at earliest levels of learning

Encounter	Pupils are present during an experience or activity without any obvious learning outcome, although for some pupils, *for example, those who withhold their attention or their presence from many situation*s, their willingness to tolerate a shared activity may, in itself, be significant.
Awareness	Pupils appear to show awareness, noticing that something has happened and, fleetingly focusing or attending to an object, event or person, *for example, by briefly interrupting a pattern of self-absorbed movement or vocalisation*.
Attention and response	Pupils attend and begin to respond, often not consistently, to what is happening, *for example, by showing signs of surprise, enjoyment, frustration or dissatisfactio*n, demonstrating the beginning of an ability to distinguish between different people, objects, events and places.
Engagement	Pupils show more consistent attention and can tell the difference between specific events in their surroundings, *for example, by focused looking or listening; turning to locate objects, events or people; following moving objects and events through movements of their eyes, head or other body part*s.
Participation	Pupils engage in sharing, taking turns and the anticipation of familiar sequences of events, *for example, by smiling, vocalising or showing other signs of excitemen*t, although these responses may be supported by staff or other pupils.
Involvement	Pupils actively strive to reach out, join in or comment in some way on the activity itself or on the actions or responses of the other pupils. F*or example, by making exploratory hand and arm movements, seeking eye contact with staff or other pupils, or by speaking, signing or gesturing.*
Gaining skills and understanding	Pupils gain, strengthen or make general use of their skills, and understanding knowledge, concepts or understanding that relate to their experience of the curriculum, *for example, they can recognise the features of an object and understand its relevance, significance and use.*

The framework may give staff a greater understanding of how pupils move through a learning process because it offers a commonsense perspective that illustrates a general hierarchy of learning at fundamental levels.
However Routes for Learning and Quest for learning both emphasise that though the framework describes is a linear continuum, pupils may actually follow other paths dictated by personal factors. The assessment templates offered by Routes for Learning and Quest provide flexible ways of depicting the routes that pupils take.

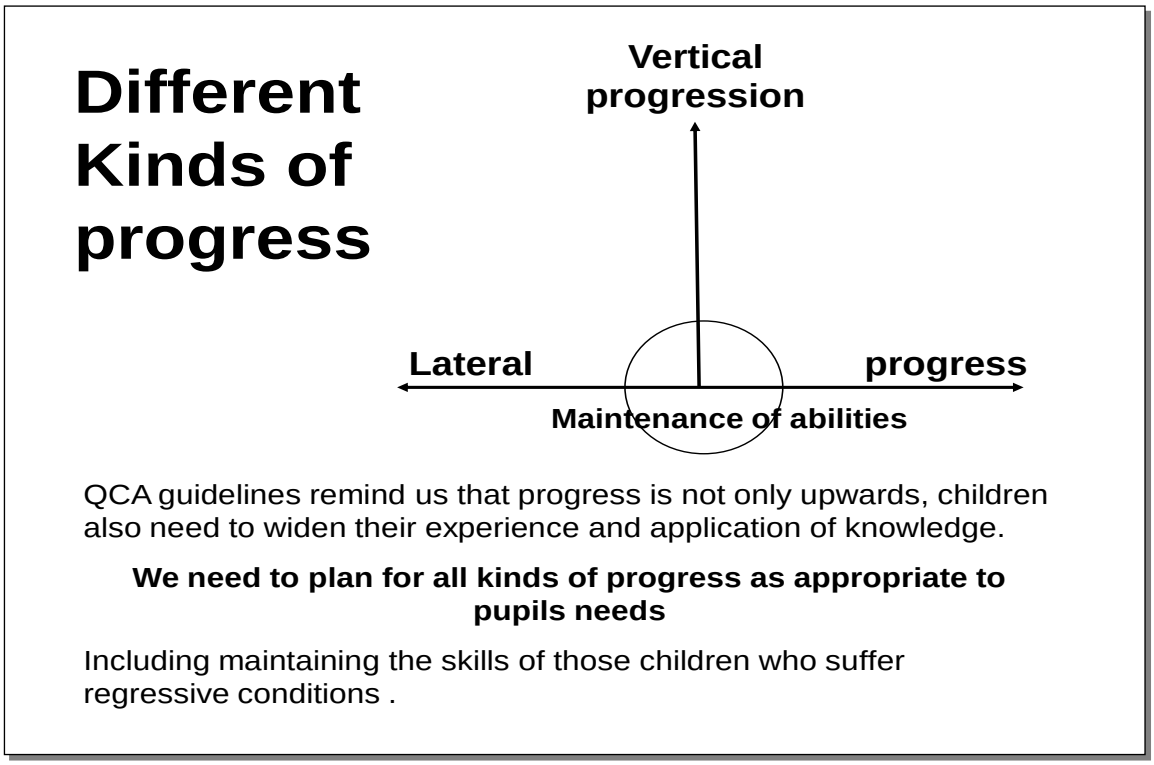
A version relating this material to English P Scales is available from the author

The figure that follows illustrates the the English P scales framework and the key words connected to different P levels. Readers will note that there is not only inherent transition within or across levels in this way we can see that pupils may spiral or work at overlapping levels. .

Different kinds of progress

During the course of their development children make progress in different ways. Sometimes they take learning steps that take them up to a next level of skill and knowledge – and these 'vertical ' steps are the ways in which the mainstream curriculum usually measures progress – **for example the P scales illustrate such a progressive ladder. However vertical steps** are not the only way that learning progresses. Sometimes people make important 'lateral' progress – w**hen they** learn to apply what **they** have learned in one situation **to other** situations and contexts. In this way we widen the application and usefulness of the learning, it is also connected to other memories, knowledge and skills and is consolidated, which in turn enables pupils to progress and spiral to subsequent vertical levels.[1]

The spontaneous and exploratory nature of children's play has an important role in such lateral learning, and in contributing to vertical progress, because it includes natural repetitions, practising physical movements, sequencing, making connections, drawing inferences and conclusions, testing rules and knowledge etc. all of which contribute to the development of skills and knowledge.



Aspects of lateral progress -- progress within levels

'Routes for learning' (Welsh Assembly 2006) describes a number of phases related to aspects of skill development. It is illustrated in the table below and provides useful perspective for us when we observe our children's activities – and ask ourselves – what's going on? – what's the nature or purpose of this play? – This in turn might help us reflect upon what we want to achieve from planned activities or interactions with pupils. E.g. setting objectives for how children may develop greater fluency, consolidate or generalise skills etc.

A range of aspects related to skill development and learning	
Maintenance	In which a child maintains their level of response with their environment. (*This may represent considerable achievement for children suffering declining conditions or temporary difficulties)*
Reactivation	In which learners regain skills apparently lost , and begin to re kindle their interest in responding to their environment
Acquisition	As learners learn new responses through demonstration, modelling or physical prompting often with great deal of support.
Fluency	Learning through repeated doing to develop greater confidence, ease of action and accuracy – approaching levels of mastery.
Consolidation	In which learners consolidate competency and fluency over time by over learning through repetition and familiarity. *E.g. They will remember how to do the task after a break.*
Generalisation	in which learners develop and achieve mastery in different settings or contexts, with different stimuli or materials or with different staff
Application and adaptation	When learners recognise similarities and difference in new situations and adapt application of skills to them

Some schools are using these aspects of skill development to help them describe pupils progress within levels.

Schema – extending reflex action into active exploration and thinking.

In the section of the EQUALS guide about teaching and learning I refer to 'schema' as initially being patterns of behaviour springing from reflexes, gradually integrating together to become useful skills. '**Physical schema',** for example – the combination of circular arm movements and the Palmar grasp being integrated to enable picking things up.
I also discuss how children continue to use combine, interlink and refine schematic skills, and how they become increasingly complex as children's exploration and interaction with the wider world progresses. I note examples like the progressive refinement of different finger and hand manipulations, or the development of casting away and retrieving etc.

However schema are not just physical actions; because as children remember:

- The physical and sensory experiences of the actions

The feeling of the arm sweeping through space- and the palm making contact with the solid object'

- They also relate consequences to actions

These memories are the beginnings of their thinking about processes and things and effects.
Their physical activities are becoming remembered and attached to other thoughts,
they are becoming a part of the Childs thinking – they are the physical embodiment of 'ideas' **– intellectual schema.**

Early years practitioners like Athey (1990)[vi] Bruce (1997)[vii] observe that there are particular patterns to the way that children use their physical schema to explore and develop ideas and in her book "Threads of thinking" (1999)[viii] Cathy Nutbrown identified a rich variety of mathematical ideas being explored by nursery-aged children through schematic activity. These included – numbers, sorting, time, matching, quantity, position, estimation, transformation, addition, length, equivalence, distance, symmetry, cause and effect, rotation and trajectory.

Schematic activities are built through combinations of a small number of major schemas.

- Vertical and horizontal
- Circular movements.
- Enveloping or containing.

As typical children practice each schema they often have an idea that is the dominant focus of their investigation, though sometimes they are interested in a cluster of different schema.
Combinations can create or represent many forms of actions and thoughts. Sometimes children flit from one activity to another but certain schematic patterns recur during different aspects of learning and when children are in different circumstances. – So for a couple of weeks the typical child might be interested in posting things into slots.

It is easy to have an initial impression that schema only applies at early levels, but they do recur in different forms, at different levels in our learning. For example visual tracking of moving objects is a skill pursued as a practical observation skill by infants, and may be a proficiency that some pupils with profound difficulties need to pursue, whilst at later levels learning to read requires a tracking of text.
This might suggest to us that :

- we can see progress as pupils mature the application of their schematic learning
- the application of schema at different levels provides us with avenues of differentiation.

As typical children become interested in drawing, aspects of schema become evident in their pictures:

- They develop skill in drawing shapes that correlate to things or people
- They experiment with position etc
- They will draw groups of people and things, or make tally marks on their pictures to record

In their excellent book ' Childrens Mathematics - making marks making meaning' . Worthington and Carruthers (2006)[ix] Discuss the development of such **representational schema** and their relationship to mathematical development and abstract thinking.

Here are some examples of activities that relate to mathematical learning, which arise from ways in which the three major schemas interact.
Many of them are related, all of them can be incorporated into many activities and most have different forms and levels of subtlety that children need to master.

- They give us back ground material for observing pupils
- They may give us ideas for setting up learning opportunities in which children can explore and develop their skills and thoughts.

Schema related to mathematical learning

Holding
To have and to hold, a powerful tactile experience, that grows from that fundamental response of the newborn child to grasp things. It may initially include holding grasping their own fingers or holding objects.

Soon holding is coordinated with looking, surely this is one of the experiences from which the concept of 'one' springs and surely its tactile messages are essential to the awareness of objects that will eventually be a constituent part of our understanding of 'object permanence' – and to exploring aspects of form and weight.

Holding is related to other enclosure and containment schema, there are many combinations and progressions from elementary holding, as new dimensions are added, two-handed picking, eye hand coordination, and hand transfer, sequential holding, holding items in each hand; all are skills offering sensory knowledge, not only about the properties of objects but also about the power to move and reorganise them.

Tracking
Tracking may be visual, tactile movement or auditory – or may involve integrated use of senses. At earliest levels tracking may be observed as fixing interest on particular items and following them, later scanning rows of objects is an important precursor to counting and the development of the internal image of a number line.

The development of tracking or tracing activities is of importance both in respect of following information, and in making kinaesthetic and tactile memory imprints e.g. sequential touching, that support or compensate mental visual imagery. So the simplest forms of tracking schema are an important first practice for more sophisticated, visual control, searching, and sequencing, as well as tracking that we use in later skills for example making letter or numeral forms, following diagrams, mazes or maps.

Alternating
Being aware of more than one item and switching attention between them. The process of alternating gaze is an early communication skill, part of eye pointing.

Alternating gaze is also first step towards maintaining attention with more than one object, and switching attention between objects, as well as an essential skill for counting to two.

Alternating holding and touching and other cross lateral activities are processes that develop understanding of sequence and stimulate cerebral activity between the two sides of the brain. We must learn to alternate between items to be able to examine and learn to match and compare them and develop all the skills that are based upon those fundamental operations.

Dabbing
A schema often used in paintings and drawings sometimes haphazardly, or with many repetitions being stimulated by the impulses of the act of making the mark. Sometimes children associate this with representing very large numbers – 'hundreds' or 'millions'.

More systematic dabs may be used to represent numbers they know for example, eyes, or animals in a field.

Some dabbing may be used to imitate patterns of small numbers related to subitising – *recognising, knowing and representing small groups.*

Pointing and itemising
Noticing one thing at once and pointing and touching, are obvious precursors to counting. There is a hierarchy in the process of learning to point ranging from simple looking and reaching or showing

behaviours, to declarative finger pointing – one might also see sequential eye scanning and nodding as a form of pointing.

Pointing has various purposes

- it is a communication skill
- it is an organisational skill for counting.

It involves not only the organisation of the child's own hands and digits but the coordination of looking. To extend pointing as a counting activity also entails developing 'alternating' and sequence schemas, requiring the child to switch attention and maintain sequences of touching. Sequential touching provides a powerful stimulus registering quantity.

Matching and one to one corresponding

Building upon other schema such as pointing and grouping children may begin to relate to the rules of one to one correspondence. They may like to match items to people, collect single items in containers, find partners to play share or work with.
There is often evidence of this schema in scrap collages and constructions where a child for example places single items inside each cup of an egg box.

Moving and transporting

However as children may move objects or collections of objects from one place to another the processes may help them to experience:

- Progressive changes of quantity .
- Conservation.
- Space, direction and position, and important opportunities to experience related language.

Seeing things change position but still remain the same. Seeing piles grow or dwindle experiencing all gone all have relations to concepts of number. Shifting items that have been pointed at one at a time, and coordinating a verbal note as each item is moved, are ways in which this schema relates to learning about counting.

- Moving items from one pile to another illustrates addition or subtraction.
- Distributing objects can relate to division.
- Collecting groups of objects together illustrates multiplication.

Collecting/grouping

Children might like to put things together collect items in their arms or containers; they may like to form and join groups of other children. Later they may like to create groups sorted by some kind of criteria. There are perceptual influences, such as similarity and proximity that affect our ideas and decisions about grouping things and these explorations give pupils experience of thinking about them and the related rules of sorting.

Like moving and transporting, these activities also have obvious connections to understanding processes of progressive increase or decrease in quantities. They also illustrate the connection of quantity to physical size.

Connecting and stacking

Children might like to connect and put things together, connect and join toys, trains bricks. Strings, rope, wool and so on may be used to tie objects together

Connecting illustrates sequencing and offers opportunities for counting illustrating the number line. Sometimes children's drawings and paintings show a series of linked parts, along with an obvious correlation to developing the understanding of number lines, and perceptual connections between number and horizontal/vertical representations of size.

Ordering and sequencing

A child may like making lines of objects, as with tracking it is a precursor to counting. They may like being beside in front or behind others, joining lines of children.
They may enjoy making sequential gestures, pointing and clapping.

Later they may produce paintings and drawings with ordered lines or dabs; collages or constructions with items of scrap carefully glued in sequence.

At more advanced stages they may show interest in 'largest' or 'smallest', organise things in order by size observing progressive change, and thinking about sorting criteria. Using bricks blocks or equipment in this way can illustrate the 'nesting' or the 'staircase image' that are intrinsic to hierarchical inclusion i.e. understanding the idea that numbers build by 'one' each time and that they nest within each other by that amount.

Separating
Children may like to take things apart and separate items. They may like to give things away, or enjoy activities where groups decrease, observing a decrease to nothing. They may like to share things, and group by separating items.

Such processes may illustrate practical subtraction or division, eventually children need to come to an understanding that separating is the inverse of collecting, and eventually subtraction is the inverse of addition.

Positioning
Children may like to place themselves in different places and positions, which help them experience that there are different viewpoints. A child may be interested in placing objects in particular positions e.g. on top of something, around the edge, behind. Paintings and drawings also often show evidence of this.

Positioning has obvious connections with understanding space, but some positioning schema may include hiding and finding which helps with growing an awareness of object permanence**.**

Orientation
Children have interest in different viewpoints as when a child enjoys turning round, finding a viewpoint, hangs upside down or turns objects upside down.

Balance
Children may be interested in balance, creating stability, symmetry, experimenting with equality and differences

Dynamic vertical and horizontal and diagonal schema
May be instrumental in and may developing ideas related to height and dimensions. A child may show evidence of particular interest by actions such as climbing, stepping-up, jumping down and lying down flat. Feeling vertical and horizontal structures, such as poles, planes such as tabletops or walls, hand rails. These schemas may also be seen in constructions, collages or graphically in drawing or painting.

Crosses and grids
After schemas of horizontality and verticality have been explored separately the two are often used in conjunction to form crosses or grids**.** These are very often systematically explored on paper and interest is shown in everyday objects such as a cooling tray, grills, nets or boxes, drawers.

Diagonals
Usually later than the vertical/horizontal schemas interest can be seen in the child's fascination with ramps, slides, stairs and stair rails. Drawings begin to contain diagonal lines forming roofs, triangles, and zigzags.

Corners and angles

Circles
Circular motions and manipulations are with the child in reflex activities from the very beginning, and later include rolling, spinning, turning, which relate to positioning and orientation schema.
Running round and circle games are part of the child's developing understanding of – completion, cycles of activity, repetition.
Circular and repeating activities are often related to counting turns and cycles.
Circles are commonly used by the child in developing symbolic representations, they appear singly or combined in drawings and paintings as heads, bodies, eyes, ears, hands, feet, flowers, wheels, the sun and a wide variety of other things. They are also related to concepts of enclosure and part of the processes of representing sets.

Rotation or rolling
There may be a fascination with turning or spinning, roll herself/himself, she may rotate her arms. A child may become absorbed by things which turn e.g. dials, taps, wheels, and keys. She may roll pencils, cylinders along, use Plasticine or dough to create sausage or balls using rotation palm movements or construct objects with rotating parts.

Radial
Circle games provide experiences of looking toward different points of the surrounding environment, building concepts of directionality. Radiating strokes are used to represent suns, fingers, eyelashes, hair.

Semi-circularity.
Semi-circles may be used in pictures or constructions to represent parts of bodies and other things. Mouths, eyebrows, ears, rainbows and umbrellas are a few of the uses for this schema as well as parts of letters or numerals.
They are part of the child's early understanding of incompleteness or fraction.

Trajectory and curves
The child may show an interest in things travelling or flying through the air e.g. balls, planes, rockets, Frisbees, whatever thing can be thrown. The schema may also be expressed through child's own body movements, such as large arm throwing movements, kicking.
Children may make arcs in sand, water or spilt food – or on drawings.
They may enjoy curving movement, swinging or sweeping, trampoline, or pool movement.

Containing and Enclosure
Is related to filling and emptying and becomes related to estimating size and volume. At its earliest this may include putting the thumb into the mouth,
An interest in placing and fitting things in enclosed spaces, may include themselves, or may include being as interested in the box as the toy – or in posting items into drawers, letter boxes or money boxes – the classic used to be your keys into the video.

A child may build enclosures with building blocks, boxes. Sometimes they are named as objects e.g. buses, houses, fields, and things or people put in them, sometimes left empty. It may become related to more and less and quantity, when groups of items are enclosed in containers drawers or drawings.

In graphic form an enclosing line is often used to frame drawings. When children are exploring this schema, sometimes they fill the enclosure or leave it empty.

Enclosure is related to drawing shapes and collection or demarcation of sets, and to inside / outside.

Enveloping
As an extension of enclosing, children may wrap things up or put them in boxes with covers or lids, they wrap themselves or others in a blanket or hide in boxes. They may enjoy going underwater or seeing others submerge.
Paintings are sometimes completely covered over with a single colour wash, turning a day picture to night.
Enveloping explores concepts of object permanence, constancy and conservation. It illustrates that things can still exist when they have gone out of sight. It can be used to challenge memory and promote the use of imagery as a memory tool.

Transformations
Children may be interested in transformations of materials, they may experiment with changes in shape, volume, working with practical materials building up knowledge that may contribute to understanding conservation i.e. that mass, volume, quantity does not change unless something is added or taken away.

Functional dependency
Although causal relationships are not fully appreciated, interest may be seen in the dependency of one function upon another, for example, pretending to turn an ignition key 'so that the engine will start'.

The mathematical dimensions of this schema include understanding that something must instigate change, or that when events happen there are consequences e.g. quantities may change. These are

fundamental appreciations that children must have if they are to realise that problems can occur and can be solved, that rules apply, not magic.

[i] QCA (2001) *Planning, teaching and assessing the curriculum for learners with learning difficulties* - General Guidelines *order ref QCQ/01/736*

[ii] Brown E.(1996) Religious Education for All. London: David Fulton

[iii] Byers R. (1996) 'Classroom Processes' in B. Carpenter, R. Ashdown and K. Bovair (eds) Enabling Access – effective teaching and learning for learners with learning difficulties. London: David Fulton.

[iv] Quest for Learning Northern Ireland Council for Curriculum, Examinations & Assessment (CCEA) http://www.nicurriculum.org.uk/inclusion_and_SEN/SEN/pmld.asp
CCEA acknowledge source material from welsh assembly materials routes for learning.

[v] Welsh Assembly Qualifications and Curriculum Group (2005) Cardiff http://wales.gov.uk/topics/educationandskills/curriculumassessment/additionaleducationalneeds/routeslearning/?lang=en

[vi] Athey, C . (1990) Extending Thought in Young Children London: Paul Chapman Publishing.

[vii] Bruce T (1997) Early Childhood Education. London Hodder and Stoughton

[viii] Nutbrown, C (1999) *Threads Of Thinking.* Paul Chapman Publishing. London

[ix] Carruthers E, Worthington M (2003) Children's mathematics – making marks making meaning. London . SAGE Publications. Paul Chapman Publishing